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Analytical method of radiation by a wave energy device with dual rectangular floating bodies

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Abstract

The system with one floating rectangular body on the free surface and one submerged rectangular body has been applied to a wave energy conversion device in water of finite depth. The radiation problem by this device on a plane incident wave is solved by the use of an eigenfunction expansion method, and a new analytical expression for the radiation velocity potential is obtained. The wave excitation force is calculated via the known incident wave potential and the radiation potential with a theorem of Haskind employed. To verify the correctness of this method, an example is computed respectively through the bound element method and analytical method. Results show that two numerical methods. are in good agreement, which shows that the present method is applicable. In addition, the trends of hydrodynamic coefficients and wave force are analyzed under different conditions by use of the present analytical method.

Key words: wave energy device, hydrodynamic coefficients, wave force, analytical method

1 Introduction

A dual rectangular structure is composed of a floating buoy on the free surface and a submerged buoy joined by an elastic pipe. In order to simplify the solution of this problem, the wave energy device is generally considered as cylinder buoy or rectangular buoy. In this paper, it is considered as rectangular buoy, and then the wave force is computed and the law of different external condition influence on the hydrodynamics coefficients is analyzed. This model can be applicable to much practical engineering. One typical case is the cage fish farming or other precious aquatic product of sea-farming, in which the device can be simply considered as a rectangular buoy. However, more significative application is to wave energy conversion device. The energy of wave power comes from the hydraulic pump system, which generate power depending on the wave force accumulated by the elastic pipe.

At present, there are much research achievements about rectangular buoy movement in water of constant depth of an infinite domain or semi-infinite domain. The research methods mainly have two main aspects: one is analytical method, and the other is numerical method. The former are employed in some investigations as follows: Black et al. (1971) applied the Schwingers variational formulation to the radiation of surface waves due to small oscillations of horizontal rectangular structures. Radiated wave amplitudes and wave forces were computed. Drimer et al. (1992) presented a simplified analytical model for a floating rectangular breakwater in water of finite depth. The hydrodynamic coefficients, exciting forces, and reflection and transmission coefficients are computed. Lee (1995) presented an analytical solution to the heave radiation problem of a rectangular structure. With an application of this solution, the incident waves, added mass, damping coefficients were calculated, and the hydrodynamic effects of the submergence and width of the structure were also discussed. Zheng et al. (2005) researched the interaction of a submerged rectangular buoy with incident linear waves on finite water depth and attained the solution of radiated potential and diffracted potential. The application of the latter is as

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follows: Hsu and Wu (1997) applied the linear water wave theory and the Boundary Element Method (BEM) to the hydrodynamic coefficient for an oscillating rectangular structure on a free surface with sidewall, and the influence factors on damping coefficient have been found. Hsu and Wu (1999) again applied the boundary element method to the scattering of water wave by a submerged horizontal plate and a submerged permeable breakwater. Sannasiraj et al. (1995) used the finite element technique to study the interaction of oblique waves with freely floating long structures. Then Sannasiraj et al. (1998) adopted a two-dimensional finite element model to study the behavior of pontoon-type floating breakwaters in beam waves, and Sannasiraj et al. (2000) again used the finite element method to study the diffraction-radiation of multiple floating structures in directional waves.

In this paper, the hydrodynamic coefficients and wave excitation force by a dual rectangular floating body have been studied by use of analytical method, which is composed of a floating rectangular body and a submerged rectangular body. Heave, sway and roll of floating body movement are discussed here. An eigenfunction expansion method is employed to study radiation problem of this dual rectangular wave device and expressions of wave excitation force and hydrodynamic coefficients are deduced. The wave exciting force obtained by the BEM method is adopted to verify the correctness of this analytical method. The influences of submerged depth of floating body on hydrodynamic coefficients are also given.

2 Mathematical model

Let a dual rectangular body wave energy device exist in an incompressible, inviscid and irrotational fluid. The device consists of two rectangular cylinders connected by an elastic pipe, which have the same size and shape in the vertical direction. Here the floating one on the free surface is called buoy 1 and the submerged one is called buoy 2. The origin of the coordinate system is at the undisturbed water surface with the positive z pointing upwardly and the positive x directing to the right. The structure is assumed infinite in the y direction, so the problem considered here is two dimensional and the motion modes only include heave, sway and roll. The geometrical shape of the structure and the Cartesian coordinate system are shown in Fig. 1. As time t can be able to separate from the mathematic equations, there exists a velocity potential Φ satisfying the Laplace equation. The two-dimensional governing equation for Φ is

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0. \tag{1}$$

For linear water waves considered here, it is customary to make the following decomposition.

$$\Phi = \Phi_{\rm i} + \Phi_{\rm d} + \sum_{L=1}^{3} \Phi_{\rm r}^{(J,L)}, \tag{2}$$

where $\Phi_{\rm i}$ is the incident wave potential; $\Phi_{\rm d}$ is the diffracted potential; $\Phi_{\rm r}^{J,L}$ is the radiated potential due to the motion of the floating body. Here L=1 stands for the heave, 2 for sway and 3 for roll. J=1, 2 represents buoy 1 and buoy 2 respectively. In water of finite depth, the incident wave potential of linear waves propagating along the x direction is

$$\Phi_{\rm i} = -\frac{{\rm i}gA}{\omega}\frac{\cosh[k(z+h_1)]}{\cosh(kh_1)}\exp({\rm i}kx), \tag{3}$$

where i=$\sqrt{-1}$; A is the incident wave amplitude; ω is the wave angular frequency; g is the gravitational acceleration; k is the wavenumber which is determined by the dispersion relation $k\tanh(kh_1) = \omega^2/g$ and h_1 is the water depth.

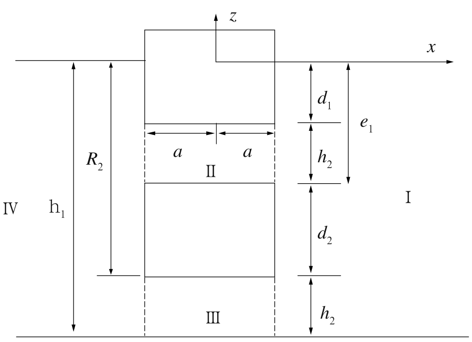

Fig.1. Schematic of the geometry.

3 Solution to the radiated potentials

3.1 *Governing equation and boundary conditions*

Let the movement of the rectangular buoy be small and the amplitude of the buoy motion be denoted by $A_{\rm r}^{(J,L)}$. The radiated potential $\Phi_{\rm r}^{(J,L)}$ by the body motion can be expressed as

$$\Phi_{\rm r}^{(J,L)} = -{\rm i}\omega A_{\rm r}^{(J,L)}\varphi_{\rm r}^{(J,L)}(x,z). \tag{4}$$

Substituting $\Phi_{\rm r}^{(J,L)}$ into Eq.(1), the following governing equations can be obtained.

$$\frac{\partial^2\varphi_{\rm r}^{(J,L)}}{\partial x^2}+\frac{\partial^2\varphi_{\rm r}^{(J,L)}}{\partial z^2}=0. \qquad (5)$$

In order to obtain the unique solution of Eq. (5), the following governing equations and the boundary conditions must be satisfied.

$$\frac{\partial\varphi_{\rm r}^{(J,L)}}{\partial z}-\frac{\omega^2}{g}\varphi_{\rm r}^{(J,L)}=0 \qquad (z=0,|x|\geqslant a), \qquad (6)$$

$$\frac{\partial\varphi_{\rm r}^{(J,L)}}{\partial z}=0 \qquad (z=-h_1), \qquad (7)$$

$$\frac{\partial\varphi_{\rm r}^{(J,L)}}{\partial z}=\delta_{J,1}[\delta_{1,L}-(x-x_0)\delta_{3,L}] \qquad (z=-d_1,|x|\leqslant a), \qquad (8)$$

$$\frac{\partial\varphi_{\rm r}^{(J,L)}}{\partial x}=\delta_{J,1}[\delta_{2,L}-(z-z_0)\delta_{3,L}] \qquad (-d_1\leqslant z\leqslant 0,x=\pm a), \qquad (9)$$

$$\frac{\partial\varphi_{\rm r}^{(J,L)}}{\partial z}=\delta_{J,2}[\delta_{1,L}-(x-x_0)\delta_{3,L}] \qquad (z=-e_1,|x|\leqslant a), \qquad (10)$$

$$\frac{\partial\varphi_{\rm r}^{(J,L)}}{\partial z}=\delta_{J,2}[\delta_{1,L}-(x-x_0)\delta_{3,L}] \qquad (z=-e_2,|x|\leqslant a), \qquad (11)$$

$$\frac{\partial\varphi_{\rm r}^{(J,L)}}{\partial x}=\delta_{J,2}[\delta_{2,L}+(z-z_0)\delta_{3,L}] \qquad (-e_2\leqslant z\leqslant -e_1,x=\pm a), \qquad (12)$$

$\varphi_{\rm r}^{(J,L)}$is outgoing at finite value when$|x|\to\propto$. (13)

Note that the coordinates (x_0,z_0) are assumed the center of rotation in Eqs (8) and (9), δ is a function introduced to express the boundary conditions simply and given by

$$\delta_{J,L}=\begin{cases}0 & J\neq L,\\ 1 & J=L.\end{cases}$$

Here the eigenfunction expansion matching method is used to solve potentials of two models. At first the fluid domain is divided into four subdomains which are I, II, III and IV as indicated in Fig. 1. The radiated potentials in four subdomains are denoted by $\varphi_{\rm r1}^{(J,L)}, \varphi_{\rm r2}^{(J,L)}, \varphi_{\rm r3}^{(J,L)}$ and $\varphi_{\rm r4}^{(J,L)}$ respectively. The method of separation of variables is applied to each subdomain in order to obtain the expressions of unknown radiated potentials. The obtained expressions in each subdomains are infinite series of orthogonal functions. They satisfy all boundary conditions but the joints of subdomain boundaries, i.e. $x=a$ and $x=-a$. The unknown coefficients in the series can be obtained by use of the continuity of pressure and velocity at these joints.

3.2 *Expressions of the radiated potentials*

Using the method of separation of variables, the space velocity potentials in each region can be expressed respectively by infinite series of orthogonal functions. In region I, the radiated potentials can be expressed as follows:

$$\varphi_{\rm r1}^{(J,L)}=\sum_{n=1}^{\propto}A_{1n}^{(J,L)}\cos[\lambda_n(z+h_1)]{\rm e}^{-\lambda_n(x-a)}. \qquad (14)$$

In region II, the radiated potentials which satisfy Eqs (1), (5) and (8) can be expressed as follows:

$$\varphi_{\rm r2}^{(J,L)}=\varphi_{{\rm r2}P}^{(J,L)}+A_{21}^{(J,L)}x+B_{21}^{(J,L)}+\sum_{n=2}^{\propto}[A_{2n}^{(J,L)}{\rm e}^{\alpha_n(x+a)}+B_{2n}^{(J,L)}{\rm e}^{-\alpha_n(x-a)}]\times\cos[\alpha_n(z+e_1)], \qquad (15)$$

where $\varphi_{{\rm r2}P}^{(J,L)}$ are particular solutions expressed as

$$\varphi_{{\rm r2}P}^{(J,L)}=\begin{cases}\frac{(z+e_1)^2-x^2}{2h_2}\delta_{J,1}-\frac{(z+d_1)^2-x^2}{2h_2}\delta_{J,2} & L=1,\\ 0 & L=2,\\ -\frac{(z+e_1)^2(x-x_0)-(x-x_0)^3/3}{2h_2}\delta_{J,1}+\frac{(z+d_1)^2(x-x_0)-(x-x_0)^3/3}{2h_2}\delta_{J,2} & L=3.\end{cases} \qquad (16)$$

In region III, the radiated potentials can be expressed as follows:

$$\varphi_{\rm r3}^{(J,L)}=\varphi_{{\rm r3}P}^{(J,L)}+A_{31}^{(J,L)}x+B_{31}^{(J,L)}+\sum_{n=2}^{\propto}[A_{3n}^{(J,L)}{\rm e}^{\beta_n(x+a)}+B_{3n}^{(J,L)}{\rm e}^{-\beta_n(x-a)}]\times\cos[\beta_n(z+h_1)] \qquad (17)$$

$$\varphi_{{\rm r3}P}^{(J,L)}=\begin{cases}\frac{(z+h_1)^2-x^2}{2h_2}\delta_{J,2} & L=1,\\ 0 & L=2,\\ -\frac{(z+h_1)^2(x-x_0)-(x-x_0)^3/3}{2h_3}\delta_{J,2} & L=3.\end{cases}$$

In region IV, the radiated potentials can be expressed as follows:

$$\varphi_{\rm r4}^{(J,L)}=\sum_{n=1}^{\propto}A_{4n}^{(J,L)}\cos[\lambda_n(z+h_1)]{\rm e}^{\lambda_n(x+a)}, \qquad (18)$$

where J=1 stands for buoy 1, and 2 for buoy 2. L always changes from 1 to 3. L=1 stands for heave, 2 for sway and 3 for roll. $A_{1n}^{(J,L)}, A_{2n}^{(J,L)}, A_{3n}^{(J,L)}, A_{4n}^{(J,L)}, B_{2n}^{(J,L)}$

and $B_{4n}^{(J,L)}$ are the unknown coefficients which need to be solved; λ_b, β_n and α_n are some eigenvalues given by

$$\lambda_1 = -\mathrm{i}k, \qquad k\tanh(kh_1) = \omega^2/g \qquad n = 1, \quad (19)$$

$$\lambda_n tan(\lambda_n h_1) = -\omega^2/g \qquad n = 2, 3, \cdots \quad (20)$$

$$\alpha_n = (n-1)\pi/h_2, \quad \beta_n = (n-1)\pi/h_3 \qquad n = 1, 2, 3, \cdots, \propto . \quad (21)$$

For the solution of these unknown coefficients above, the continuity of pressure and normal velocity at $x = \pm a$ and the eigenfunction expansion matching method are imposed on these different motion modes of each buoy. The continuity conditions are integrated in the considerate interval after they multiply by the corresponding eigenfunction on both sides, so that these conditions can be satisfied in the z direction. For simplifying the computation, the first N terms of the infinite series are intercepted, so $6N$ complex equations and the same number of unknown coefficients are obtained. These equations can be expressed as matrix equations.

$$SX_{\mathrm{r}} = F_{\mathrm{r}}^{(J,L)} \quad (22)$$

where

$$X_{\mathrm{r}} = [A_{11}^{(J,L)}, \cdots, A_{1n}^{(J,L)}, A_{21}^{(J,L)}, \cdots, A_{2n}^{(J,L)}, A_{31}^{(J,L)}, \cdots, A_{31}^{(J,L)}, \cdots, A_{3n}^{(J,L)}, A_{41}^{(J,L)}, \cdots, A_{4n}^{(J,L)}, B_{21}^{(J,L)}, \cdots, B_{2n}^{(J,L)}, B_{31}^{(J,L)}, \cdots, B_{31}^{(J,L)}, \cdots, B_{3n}^{(J,L)}]^T;$$

S is the radiated coefficient matrix; $F_{\mathrm{r}}^{(J,L)}$ is the right-hand vectors; the expressions for the elements S and $F_{\mathrm{r}}^{(J,L)}$ are illustrated in appendix A.

The linear equations are solved and the vector X_{r} can be given. So the radiated potential at any position in the fluid can be computed and the wave forces can be calculated.

4 Computation of the wave force and hydrodynamic coefficients

4.1 *Computation of wave excitation force*

The wave exciting forces result from the incident potential Φ_{i} and diffracted potential Φ_{d}.The relation of the fluid dynamic pressure to the velocity potential can be obtained from the Bernoulli equation. The dynamic pressure is integrated along the submerged surface of floating body and the wave exciting force of buoy J in direction K is $F_{kt}^j = F_k^j \mathrm{e}^{-\mathrm{i}\omega t}$. In the frequency domain, this force can be expressed as

$$F_k^j = \rho \mathrm{i}\omega \int_{S_j} (\Phi_{\mathrm{i}} + \Phi_{\mathrm{d}}) n_k \mathrm{d}s. \quad (23)$$

With an application of the Greens second identity, the Eq. (23) can also be written as

$$F_k^j = \rho \mathrm{i}\omega \int_{S_j} \Phi_{\mathrm{i}} n_k \mathrm{d}s - \rho \mathrm{i}\omega \int_{S_0} \varphi_{\mathrm{r}}^{(k)} \frac{\partial \Phi_{\mathrm{i}}}{\partial n} \mathrm{d}s, \quad (24)$$

where $S_J(J{=}1{,}2)$ is the submerged surface of floating body; and S_0 is all of the fluid boundary; n_k is the generalized normal vector outward the fluid with $n_1 = n_z, n_2 = n_x$ and $n_3 = (z - z_0)n_x - (x - x_0)n_z$, n_x and n_z are the components of unit normal vector inward the floating body. The computation methods given by Eqs (23) and (24) are respectively called approach 1 and approach 2. The analytical solution of radiated potentials is an emphasis here, so the wave exciting force is solved through the approach 2.

4.2 *Computation of radiation force*

The radiation force due to the body's motion can be calculated by the following expression:

$$F_{\mathrm{r}}^{(I,K)} = \rho \mathrm{i}\omega \int_{SI} \sum_{L=1}^{3} \sum_{J=1}^{2} \Phi_{\mathrm{r}}^{(J,L)} \mathrm{e}^{-\mathrm{i}\omega t} n_K \mathrm{d}s =$$

$$\mathrm{e}^{-\mathrm{i}\omega t} \sum_{L=1}^{3} \sum_{J=1}^{2} [\omega^2 A_{\mathrm{r}}^{(J,L)} Ca_{(I,K)}^{(J,L)} + \mathrm{i}\omega A_{\mathrm{r}}^{(J,L)} Cd_{(I,K)}^{(J,L)}],$$

where $Ca_{(I,K)}^{(J,L)}$ and $Cd_{(I,K)}^{(J,L)}$ represent the added mass and damping coefficient of buoy I in direction K due to the motion mode L of buoy J.

$$Ca_{(I,K)}^{(J,L)} = \rho \int_{SI} \mathrm{Re}(\varphi_{\mathrm{r}}^{(J,L)}) n_K \mathrm{d}s = \mathrm{Re}[\rho f_{(I,K)}^{(J,L)}],$$

$$Cd_{(I,K)}^{(J,L)} = \rho\omega \int_{SI} \mathrm{Im}(\varphi_{\mathrm{r}}^{(J,L)}) n_K \mathrm{d}s = \mathrm{Im}(\rho\omega f_{(I,K)}^{(J,L)}),$$

where $f_{(I,K)}^{(J,L)} = \int_{SI} (\varphi_{\mathrm{r}}^{(J,L)}) n_K \mathrm{d}s \qquad (L = 1, 2, 3).$

5 Results and discussions

5.1 *Verification of the present method*

To verify the correctness of the computational method in this paper, here a specific example is presented. Its geometrical parameters are $d_1/h_1 = 0.1, a/h_1 = 0.2, h_2/h_1 = 0.1, d_2/d_1 = 2$. The boundary element method and the eigenfunction expansion matching method are applied to this example respectively. Their results agree very well presented in Fig.2. The wave exciting forces of dual rectangular floating body are computed by use of the analytical method. The variation of wave exciting force and hydrodynamic coefficients including the added mass and damping coefficients are studied for different conditions. In this computation, the first 40 terms in the infinite series of the radiated potential is taken and the wave force computed by use of boundary element method for the same condition, where every computation domain is divided by 100 even units in a wavelength. $W_0 = 2\rho ga$ and $W_1 = 2\rho ga^2$ are dimensionless modules of wave exciting forces, $M_0 = 2\rho gad_1$ and $M_1 = 2\rho gad_2$ are dimensionless added mass and damping coefficients. $Ca(J,L)$ and $Cd(J,L)$ stand for the dimensionless added mass $Ca^{(J,L)}_{(I,K)}/M_0$ and damping coefficients $Cd^{(J,L)}_{(I,K)}/M_1$ respectively when the motion of buoy J is the mode L. Here only the case of $(I = J, K = L)$ is studied, namely $Ca(J,L)$=$Ca^{(J,L)}_{(J,L)}/M_0$ and $Cd(J,L)$=$Cd^{(J,L)}_{(J,L)}/M_1$. Coordinates x_0, z_0 are the reference point of roll torque and is let be the origin (0,0) and the dimensionless kh_1 is to multiply wave number by water depth.

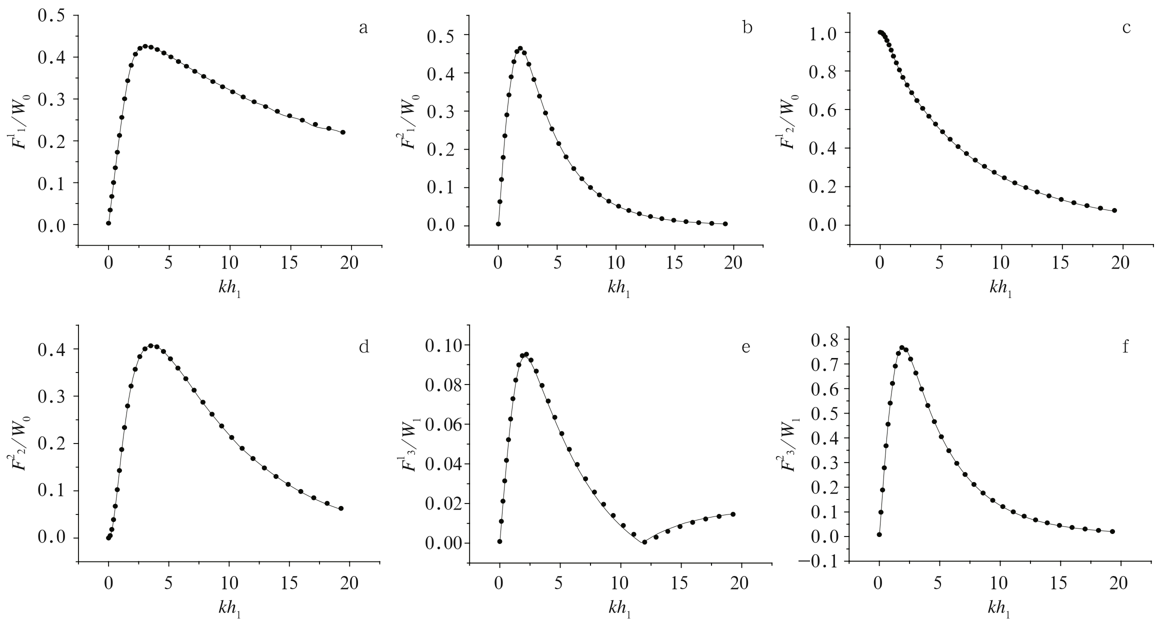

Fig.2. Comparison of the dimensionless wave forces (— analytical method; • BEM).

5.2 *Study on the hydrodynamic coefficients*

5.2.1 *A case of changing of the draft depth d_1 of buoy 1*

The variations of wave exciting forces and hydrodynamic coefficients are shown in Fig. 3 for the case in which the draft depth of buoy 1 are changed and the submerged depth of buoy 2 is not changed. It can be seen that there is almost no influence on the wave forces of buoy 1 and buoy 2 for the changed structure ratio d_1/h_1 when wave frequency is very small or very large. For the heave of buoy 1 or buoy 2, the larger the ratio of the draft to water depth is, the smaller the values of wave exciting forces and damping coefficients are and the larger the added mass is. For their sway, the larger the ration d_1/h_1 is, the larger the value of wave forces is. When the buoy 1 and 2 roll, variations of the added mass and damping coefficient are complex very much, but they only change remarkably in some specific range of frequency and almost unchanged when the wave frequency is very small or large.

5.2.2 *A case of changing of the water depth h_2 of submerged buoy 2*

Figure 4 shows the results in the case of changing of the water depth h_2 of submerged buoy 2. When the ratio of h_2/h_1 increases, the wave exciting force of buoy 1 almost no changes, but the wave exciting force of buoy 2 decreases, whose variation is large in some specific frequency range and very little in the other lower or higher frequency range. For the heave of buoy 1, with the growth of the ratio of h_2/h_1, the added mass deceases, but damping coefficient changes very slightly. For the sway of buoy, the added mass and damping coefficient are basically not affected by the submerged water depth of buoy 2. With the growth of the submerged depth h_2 of buoy 2, influence of surface

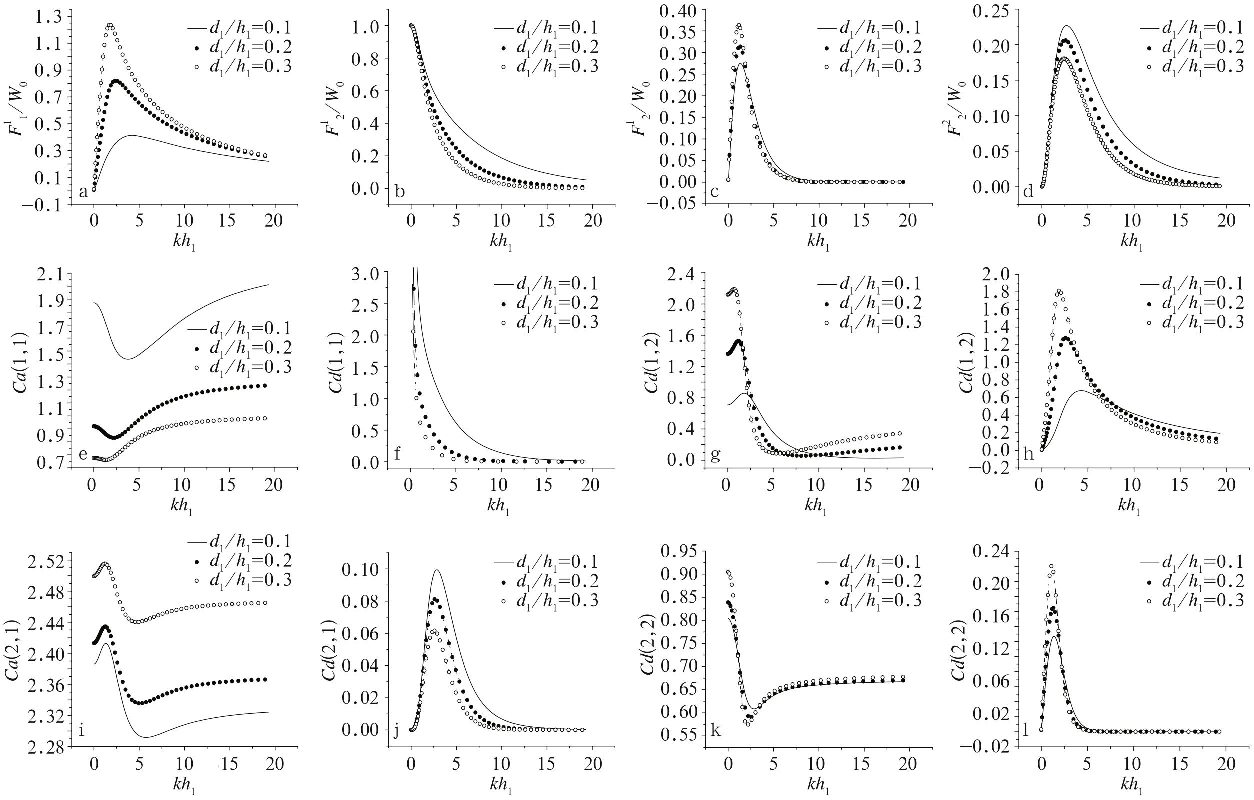

Fig.3. Comparison of the dimensionless wave forces, added mass, damping coefficient.

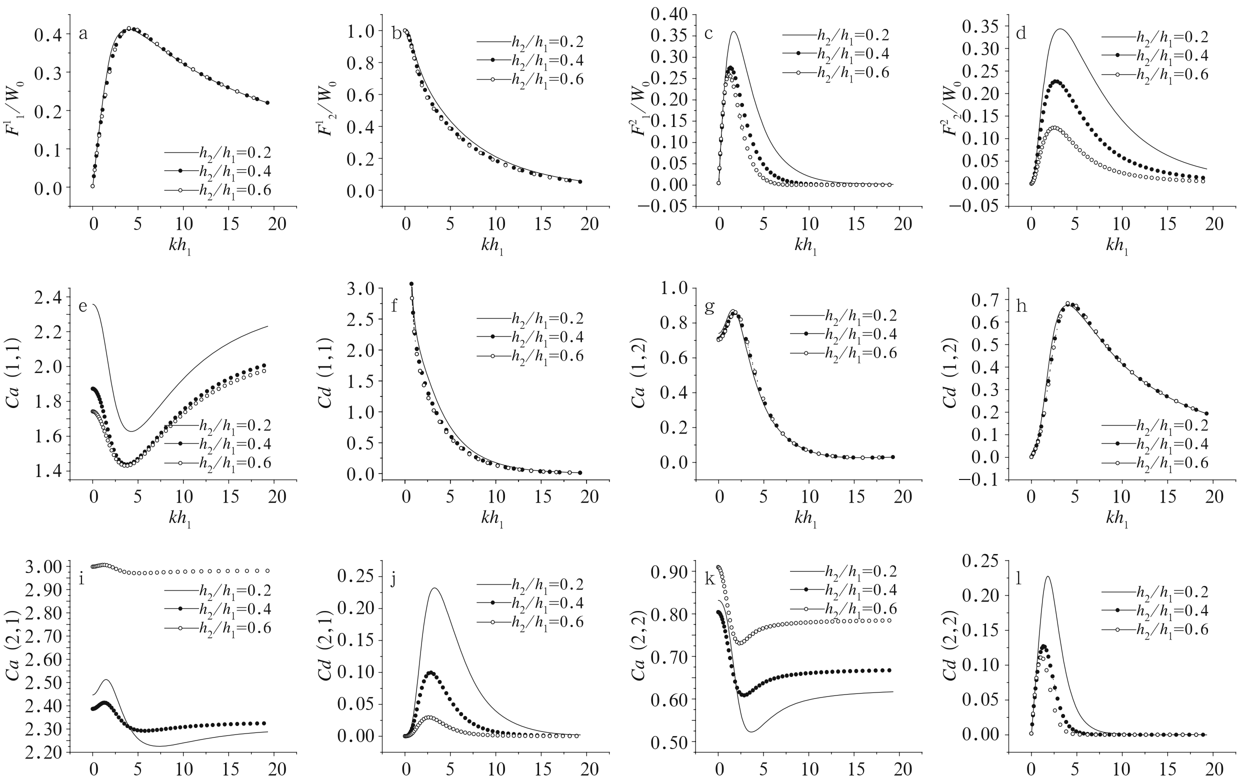

Fig.4. Comparison of the dimensionless wave forces, added mass, damping coefficient.

wave on this buoy will decrease. So the corresponding added mass and damping coefficient change with the variation of h_2. In a desirable range of frequencies the larger submerged water depth h_2 is, the bigger the added mass is and the smaller damping coefficient is. But there is almost no change for these parameters at a higher or lower frequency.

6 Conclusions

The hydrodynamic response of wave energy device with dual rectangular floating bodies is studied. The eigenfunction expansion method is employed to study radiation problem of this device. Analytical expressions of wave excitation force and hydrodynamic coefficients are deduced by use of this analytical method. This method is verified by the BEM method. For the heave, sway and roll of floating bodies, the influences of their submerged depth on hydrodynamic coefficients are also discussed. It can be concluded as follows:

(1) The analytical model of radiation by a dual rectangular floating body is applicable.

(2) The variation of submerged water depth of the top floating body has a greater influence on wave exciting forces, added mass, damping coefficients of the system.

(3) The variation of submerged water depth of the bottom floating body almost has no influence on wave exciting forces for the top floating body and its sway, but a great influence on added mass, damping coefficients for the bottom floating body.

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Appendix A. Expressions for S and F_R

(1) Expressions for the elements in matrix S

$$S_{i,j} = -\lambda_i N(\lambda_i) \tag{A1}$$

$$S_{i,N+j} = \begin{cases} -F(\lambda_i, \alpha_j) & j = 1 \\ -\alpha_j e^{2\alpha_j a} F(\lambda_i, \alpha_j) & j = 2, 3, \cdots \end{cases} \tag{A2}$$

$$S_{i,2N+j} = \begin{cases} -E(\lambda_i, \beta_j) & j = 1 \\ -\beta_j e^{2\beta_j a} E(\lambda_i, \beta_j) & j = 2, 3, \cdots \end{cases} \tag{A3}$$

$$S_{i,4N+j} = \begin{cases} 0 & j = 1 \\ \alpha_j F(\lambda_i, \alpha_j) & j = 2, 3, \cdots \end{cases} \tag{A4}$$

$$S_{i,5N+j} = \begin{cases} 0 & j = 1 \\ \beta_j E(\lambda_i, \beta_j) & j = 2, 3, \cdots \end{cases} \tag{A5}$$

$$S_{N+i,N+j} = \begin{cases} -F(\lambda_i, \alpha_j) & j = 1 \\ -\alpha_j F(\lambda_i, \alpha_j) & j = 2, 3, \cdots \end{cases} \tag{A6}$$

$$S_{N+i,2N+j} = \begin{cases} -E(\lambda_i, \alpha_j) & j = 1 \\ -\beta_j E(\lambda_i, \beta_j) & j = 2, 3, \cdots \end{cases} \tag{A7}$$

$$S_{N+i,3N+i} = \lambda_i N(\lambda_i) \tag{A8}$$

$$S_{N+i,4N+j} = \begin{cases} 0 & j = 1 \\ \alpha_j e^{2\alpha_j a} F(\lambda_i, \alpha_j) & j = 2, 3, \cdots \end{cases} \tag{A9}$$

$$S_{N+i,5N+j} = \begin{cases} 0 & j = 1 \\ \beta_j e^{2\beta_j a} E(\lambda_i, \beta_j) & j = 2, 3, \cdots \end{cases} \tag{A10}$$

$$S_{2N+i,j} = F(\lambda_j, \alpha_i) \tag{A11}$$

$$S_{2N+i,N+i} = \begin{cases} -h_2 a & i = 1 \\ -\dfrac{h_2}{2} e^{2\alpha_i a} & i = 2, 3, \cdots \end{cases} \tag{A12}$$

$$S_{2N+i,4N+i} = \begin{cases} -h_2 & i = 1 \\ -\dfrac{h_2}{2} & i = 2, 3, \cdots \end{cases} \tag{A13}$$

$$S_{3N+i,j} = E(\lambda_j, \beta_i) \tag{A14}$$

$$S_{3N+i,2N+i} = \begin{cases} -h_3 a & i = 1 \\ -\dfrac{h_3}{2} e^{2\beta_i a} & i = 2, 3, \cdots \end{cases} \tag{A15}$$

$$S_{3N+i,5N+i} = \begin{cases} -h_3 & i = 1 \\ -\dfrac{h_3}{2} & i = 2, 3, \cdots \end{cases} \tag{A16}$$

$$S_{4N+i,N+i} = \begin{cases} -h_2 a & i = 1 \\ -\dfrac{h_2}{2} & i = 2, 3, \cdots \end{cases} \tag{A17}$$

$$S_{4N+i,3N+j} = F(\lambda_j, \alpha_i) \tag{A18}$$

$$S_{4N+i,N+i} = \begin{cases} -h_2 & i = 1 \\ -\dfrac{h_2}{2} e^{2\alpha_i a} & i = 2, 3, \cdots \end{cases} \tag{A19}$$

$$S_{5N+i,2N+i} = \begin{cases} -h_3 a & i = 1 \\ -\dfrac{h_3}{2} & i = 2, 3, \cdots \end{cases} \tag{A20}$$

$$S_{5N+i,3N+j} = E(\lambda_j, \beta_i) \tag{A21}$$

$$S_{5N+i,5N+i} = \begin{cases} -h_3 & i = 1 \\ -\dfrac{h_3}{2} e^{2\beta_i a} & i = 2, 3, \cdots \end{cases} \tag{A22}$$

where :
$$N(\lambda_i) = \begin{cases} \dfrac{h_1}{2} + \dfrac{\sinh(2kh_1)}{4k} & i = 1 \\ \dfrac{h_1}{2} + \dfrac{\sin(2\lambda_i h_1)}{4\lambda_i} & i = 2, 3, \cdots \end{cases} \tag{A23}$$

$$E(\lambda_i, \beta_j) = \begin{cases} \dfrac{(-1)^{j-1} k \sinh(kh_3)}{k^2 + \beta_j^2} & i = 1 \\ \dfrac{(-1)^{j-1} \lambda_i \sin(\lambda_i h_3)}{\lambda_i^2 - \beta_j^2} & i = 2, 3, \cdots \end{cases} \tag{A24}$$

$$F(\lambda_i, \alpha_j) = \begin{cases} k\dfrac{(-1)^{j-1}\sinh[k(h_1 - d_1)] - \sinh[k(h_1 - e_1)]}{k^2 + \alpha_j^2} & i = 1 \\ \lambda_i \dfrac{(-1)^{j-1}\sin[\lambda_i(h_1 - d_1)] - \sin[\lambda_i(h_1 - e_1)]}{\lambda_i^2 - \alpha_j^2} & i = 2, 3, \cdots \end{cases} \tag{A25}$$

(2) Expressions for the elements in matrix $F_{\mathrm{R}}^{(J,L)}$

$$F_{\mathrm{r},i}^{(J,L)} = P_{1i}^{(J,L)} \tag{A26}$$

$$P_{1i}^{(J,1)} = \int_{-e_1}^{-d_1} \frac{a}{h_2}[-\delta_{J,1} + \delta_{J,2}]\cos[\lambda_i(z+h_1)]\mathrm{d}z + \int_{-h_1}^{-e_2} \frac{-a}{h_3} - \delta_{J,2}\cos[\lambda_i(z+h_1)]\mathrm{d}z$$

$$P_{1i}^{(J,2)} = \int_{-d_1}^{0} \delta_{J,1}\cos[\lambda_i(z+h_1)]\mathrm{d}z + \int_{-e_2}^{-e_1} \delta_{J,2}\cos[\lambda_i(z+h_1)]\mathrm{d}z$$

$$P_{1i}^{(J,3)} = \int_{-d_1}^{0} \delta_{J,1}(z-z_0)\cos[\lambda_i(z+h_1)]\mathrm{d}z + \int_{-e_2}^{-e_1} \delta_{J,2}(z-z_0)\cos[\lambda_i(z+h_1)]\mathrm{d}z$$

$$F_{\mathrm{R},N+i}^{(J,L)} = P_{2i}^{(J,L)} \tag{A27}$$

$$P_{2i}^{(J,1)} = -P_{1i}^{(J,1)}$$

$$P_{2i}^{(J,2)} = P_{1i}^{(J,2)}$$

$$P_{2i}^{(J,3)} = \int_{-d_1}^{0} \delta_{J,1}(z-z_0)\delta_{3,L}\cos[\lambda_i(z+h_1)]\mathrm{d}z + \int_{-e_2}^{-e_1} \delta_{J,2}[(z-z_0)\delta_{3,L}]\cos[\lambda_i(z+h_1)]\mathrm{d}z$$

$$F_{\mathrm{R},2N+i}^{(J,L)} = P_{3i}^{(J,L)} \tag{A28}$$

$$P_{3i}^{(J,1)} = \int_{-e_1}^{-d_1} [\frac{(z+e_1)^2 - a^2}{2h_2}\delta_{J,1} - \frac{(z+d_1)^2 - a^2}{2h_2}\delta_{J,2}]\cos[\alpha_i(z+e_1)]\mathrm{d}z$$

$$P_{3i}^{(J,2)} = 0$$

$$P_{3i}^{(J,3)} = \int_{-e_1}^{-d_1} [-\frac{(z+e_1)^2(a-x_0) - (a-x_0)^3/3}{2h_2}\delta_{J,1} + \frac{(z+d_1)^2(a-x_0) - (a-x_0)^3/3}{2h_2}\delta_{J,2}]\cos[\alpha_i(z+e_1)]\mathrm{d}z$$

$$F_{\mathrm{R},3N+i}^{(J,L)} = P_{4i}^{(J,L)} \tag{A29}$$

$$P_{4i}^{(J,1)} = \int_{-h_1}^{-e_2} \frac{(z+h_1)^2 - a^2}{2h_3}\delta_{J,2}\cos[\beta_i(z+h_1)]\mathrm{d}z$$

$$P_{4i}^{(J,2)} = 0$$

$$P_{4i}^{(J,3)} = -\int_{-h_1}^{-e_2} \frac{(z+h_1)^2(a-x_0) - (a-x_0)^3/3}{2h_3}\delta_{J,2}\cos[\beta_i(z+h_1)]\mathrm{d}z$$

$$F_{\mathrm{R},4N+i}^{(J,L)} = P_{5i}^{(J,L)} \tag{A30}$$

$$P_{5i}^{(J,1)} = P_{3i}^{(J,1)}$$

$$P_{5i}^{(J,2)} = P_{3i}^{(J,2)}$$

$$P_{5i}^{(J,3)} = -\int_{-e_1}^{-d_1} [\frac{-(z+e_1)^2(a+x_0) + (a+x_0)^3/3}{2h_2}\delta_{J,1} - \frac{-(z+d_1)^2(a+x_0) + (a+x_0)^3/3}{2h_2}\delta_{J,2}]\cos[\alpha_i(z+e_1)]$$

$$F_{\mathrm{R},5N+i}^{(J,L)} = P_{6i}^{(J,L)} \tag{A31}$$