A HYDRODYNAMIC MODEL OF A CIRCULATING FLUIDISED BED WITH LOW-DENSITY PARTICLES

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In this study, a two-dimensional mathematical model was developed considering the hydrodynamic behaviour of a circulating fluidised bed biomass gasifier (CFBBG), which is also applicable for other low-density particles. In the modelling, the CFB riser was divided into two regions: a dense region at the bottom and a dilute region at the top of the riser. Kunii and Levenspiel's [Kunii and Levenspiel, Powder Technol. 61, 193-206 (1990)] model was adopted to express the vertical solids distribution with some other assumptions. Radial distributions of bed voidage were taken into account in the upper zone by using Zhang et al.'s [Zhang et al., Chem. Eng. Sci. 46(12), 3045-3052 (1991)] correlation. For model validation purposes, a cold model CFB was employed, in which sawdust was transported with air as the fluidising agent. The column is 10 m in height and 280 mm in diameter, and is equipped with pressure transducers to measure axial pressure profile and with a reflective optical fibre probe to measure local solids holdup. A satisfactory agreement between the model predictions and experimental data was found.

Keywords: hydrodynamics, circulating fluidised bed, low-density particles, modelling

INTRODUCTION

The hydrodynamics of the gas–solid flow structure in circulating fluidised beds (CFB) are of dominant importance for predicting the conversion of CFB combustors/gasifiers. The flow structure of gas–solid flows in a CFB is inherently very complex. Up to now, considerable work has been done on the investigation of hydrodynamics in CFB reactors (Grace et al., 1997).

Modelling of CFB is rather difficult. Therefore, it is necessary to develop simplified modelling approaches, which can describe the gas–solid flow structure with sufficient accuracy. Many modelling efforts with various assumptions and different mathematical formulations have been reported in literature. Harris and Davidson (1994) have classified all models in three types: (I) those that predict axial variation in solids suspension density, but not radial variations (Li and Kwauk, 1980; Rhodes and Geldart, 1987; Kunii and Levenspiel, 1991; Smolders and Baeyens, 2001); (II) those that predict both axial and radial variations by assuming two or more regions, for example core–annulus flow structure or clustering annular flow models (Rhodes, 1990; de Diego et al., 1995; Nieuwland et al., 1996; Pugsley and Berruti, 1996; Gungor and Eskin, 2007); (III) those that employ the fundamental equations of fluid dynamics to predict two-phase gas–solid flow (Sun and Gidaspow, 1999; Mathiesen et al., 2000; Cabezas-Gómez and Milioli, 2003; Lu and Gidaspow, 2003; Wang et al., 2008). Type I models are oversimplified to provide rough predictions. The radial variations are completely neglected, which seriously limited the ability of the models to represent CFB reactors. Type III models may be the most rigorous, but the required simplifying assumptions combined with their mathematical complexity limit their use to studies of specific flow structures within the riser. Type II models provide more details concerning the radial distribution of solids and allow one to investigate the flow structure of CFB risers in a two-dimensional way.

Rhodes (1990) proposed a core–annulus model for the determination of the internal solid flow structure in a CFB. The model permits prediction of the form of the radial solids flux profile and of the development of this profile with changing axial position. de Diego et al. (1995) proposed two empirical equations for the

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calculation of the solid flux arising from the dense region as a function of the operating conditions. Pugsley and Berruti (1996) developed a predictive hydrodynamic model to describe both axial and radial flow structure in CFB risers. The model postulates the existence of a core–annulus type of flow structure and is based on both fundamental principles and empirical relationships.

In this study, a two-dimensional mathematical model is developed to predict both axial and radial solids holdup and bed voidage profiles of a cold model CFB riser with sawdust particles. Experiments are performed to verify the simulation results.

CFB MODEL

Based on the experimental findings in our lab and in the literature, a two-dimensional mathematical model was developed considering the hydrodynamic behaviour of large-scale CFB risers. The main assumptions of the proposed model are:

- the riser is axially divided into two regions: a dense region at the bottom and a dilute region at the top of the riser;
- the dense region, if it exists, has a constant solids holdup;
- in the dilute upper region solids holdup varies both axially and radially;
- on a time averaged scale, any nonuniformity caused by clusters or packets of solids is neglected;
- exit effects are neglected.

Vertical Profile of Solids Holdup

Several approaches have been proposed to investigate vertical profile of solids distribution in CFB risers (Li and Kwauk, 1980; Wen and Chen, 1982; Rhodes and Geldart, 1987). Based on the observations that the solids concentration decays exponentially from the bed surface, Kunii and Levenspiel (1990) developed a model to describe backmixing in the freeboard above the bubbling bed. The model was also suggested to be valid for circulating fluidised beds (Kunii and Levenspiel, 1991). This model is modified and applied in the present study and the vertical solids holdup distribution is expressed as:

$$\frac{\varepsilon_s - \varepsilon_s *}{\varepsilon_{sb} - \varepsilon_s *} = \exp[-a(z - H_b)] \qquad (1)$$

where ε_{sb} is the solids holdup at the dense region, H_b is the height of the dense region and is experimentally determined. ε_s^* is the solids holdup in the fully developed dilute region. Decay constant a is a function of particle properties and operating conditions, and the following equation was recommended by Adánez et al. (1994) to calculate the decay constant:

$$a(u_g - u_t)^2 = 3.5 - 1670 d_p \qquad (2)$$

A large amount of studies have been done experimentally and theoretically regarding ε_{sb} and ε_s^*. Kunii and Levenspiel (1991) summarised a large amount of experimental findings and compared bottom bed solids holdup ε_{sb} in various fluidising regimes. They gave a range of $\varepsilon_{sb} = 0.16$–0.22 for fast fluidisation regime. Schlichthaerle and Werther (1999) studied solids concentration distribution in the bottom zone by performing the experiments in a cold model CFB unit with a diameter of 0.4 m and a height of 15.6 m. Operating conditions typical of CFB combustors were chosen ($u_g = 3$–5 m/s, $G_s = 5$–$50\ \mathrm{kg/m^2\,s}$). Quartz sand ($d_p = 65$–$340\ \mu\mathrm{m}$) was used in the experiments. A solids holdup of $\varepsilon_{sb} = 0.2$–0.3 was observed. Svensson et al. (1996) investigated fluidisation regimes of the bottom bed of Chalmers 12 MWth CFB boiler and a cold model CFB. They used sand as their bed material ($d_p = 190$–$430\ \mu\mathrm{m}$, $\rho_p = 2600\ \mathrm{kg/m^3}$), and operating conditions $u_g = 0.4$–6.5 m/s, $G_s \sim 10\ \mathrm{kg/m^2\,s}$. The average solid holdup of the bottom bed was obtained from pressure drop measurements and $\varepsilon_{sb} \approx 0.35$–$0.45$. This range was larger than that of Kunii and Levenspiel's (1991), which may be due to a fairly large almost square combustor ($1.47 \times 1.42 \times 13.5\ \mathrm{m^3}$) was employed and the superficial gas velocity was relatively low. Bai and Kato (1999) provided a wide range of summarisation of empirical correlations in the open literature and proposed two more generalised correlations for both ε_{sb} and ε_s^* based on experimental data from the literature and their laboratory. These correlations, however, were later shown to be experimentally invalid for coarse and light biomass particles (e.g. cork) by Mei et al. (2006).

Since there are very few models concerning fluid structure of low-density particles in CFBs, the following correlations are proposed by using least squares regression method for ε_{sb} and ε_s^* based on experimental data from the literature and our laboratory:

$$\varepsilon_{sb} = 1.0422\left(\frac{G_s}{\rho_p(u_g - u_t)}\right)^{0.2781}\left(\frac{u_g}{\sqrt{gD}}\right)^{-0.3218}\left(\frac{d_p \rho_g u_t}{\mu}\right)^{-0.1195} \qquad (3)$$

$$\varepsilon_s^* = 0.0104\left(\frac{G_s}{\rho_p(u_g - u_t)}\right)^{1.2933}\left(\frac{u_g}{\sqrt{gD}}\right)^{0.0808} Ar^{0.9822} \qquad (4)$$

valid for $d_p > 200\ \mu\mathrm{m}$, $\rho_p < 800\ \mathrm{kg/m^3}$, $u_g = 1.5$–6 m/s, $G_s = 1$–$150\ \mathrm{kg/m^2\,s}$.

The experimental data used to correlate the solids holdups are listed in Table 1. We are mainly focused on those low-density and large particles ($d_p > 100\ \mu\mathrm{m}$, $\rho_p < 800$), for example biomass particles and silica gel, because we believe these same type of particles should have some hydrodynamic characteristics in common.

The comparisons between predicted and experimental values for ε_{sb} and ε_s^* are shown in Figures 1 and 2, respectively. The predicted and the measured values matched pretty well within 20% of relative error.

Radial Distribution of Bed Voidage

Extensive researches have been conducted in recent decades to measure and characterise the radial profiles of particle concentration (Zhang et al., 1991; Rhodes et al., 1992; Patience and Chaouki, 1995; Godfroy et al., 1999; Xu et al., 2004). The radial profile which had been observed in small-scale equipment can be represented by a core–annulus flow structure in the radial direction. The validity of this model has been confirmed by experimental observations in a large-scale CFB (Werther, 1994).

Zhang et al. (1991) reported radial void fraction profiles for four different powders in three different risers up to 0.3 m in diameter. They found that the normalised radial voidage was a unique function of radial position, r/R, based solely on the cross-sectional average voidage, $\bar{\varepsilon}$:

$$\varepsilon = \bar{\varepsilon}^{(0.191 + (r/R)^{2.5} + 3(r/R)^{11})} \qquad (5)$$

A correlation of voidage based only on cross-sectional average voidage is reasonable because radial voidage profiles under different operating conditions (riser radius, solids flux, and superficial gas velocity) are similar, as shown by Zhang et al. (1991). Such a correlation is convenient because a single measurement or prediction of cross-sectional average voidage can be used to map

Table 1. Data used to correlate solids holdups

Symbol	Particle	d_p (μm)	ρ_p (kg/m^3)	D (m)	u_g (m/s)	G_s (kg/m^2 s)	ε_sb	ε_s^*	Ref.
○	Silica gel	280	706	0.186	3.7	84.4	0.18	0.09	Bi et al. (1989)
○	Silica gel	280	706	0.186	6	93.9	0.1	0.05	
○	Silica gel	280	706	0.186	6	133.8	0.199	0.07	
×	Silica gel	220	794	0.115	5.3	132	0.2	0.05	Yang et al. (1984)
×	Silica gel	220	794	0.115	5.3	160	0.25	0.057	
*	Silica gel	205	760	0.09	4.25	82	0.21	—	Gao et al. (1991)
*	Silica gel	205	760	0.09	3.37	60	0.212	—	
*	Silica gel	205	760	0.09	2.7	37.3	0.225	—	
Δ	Cork	812	189	0.305	3.2	3	0.17	0.06	Mei et al. (2006)
◊	Sawdust	400	430	0.28	2	1.12	0.11	0.003	This work

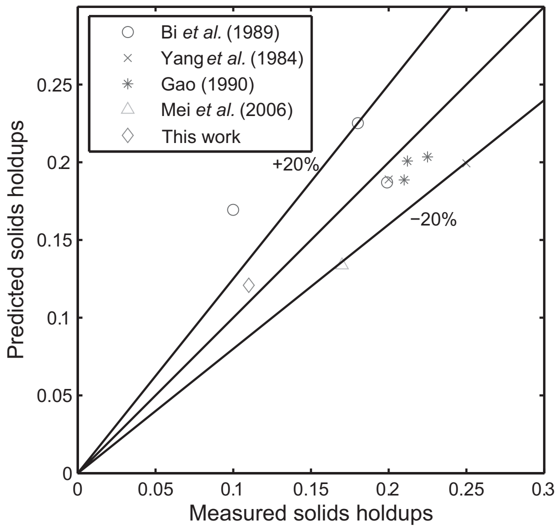

Figure 1. Comparison between prediction and measurement (ε_sb). [Color figure can be seen in the online version of this article, available at http://onlinelibrary.wiley.com/journal/10.1002/(ISSN)1939-019X]

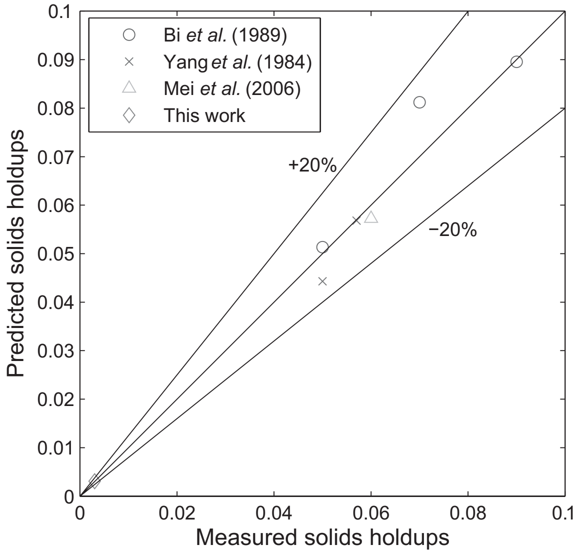

Figure 2. Comparison between prediction and measurement (ε_s^*). [Color figure can be seen in the online version of this article, available at http://onlinelibrary.wiley.com/journal/10.1002/(ISSN)1939-019X]

voidage at a given cross-section of the riser, rather than a three-dimensional model. Therefore, this equation is employed in the present model.

EXPERIMENTAL

To correlate the proposed equations (Equations 3 and 4) and validate the developed model, experiments were performed in a cold model circulating fluidised bed to investigate the effect of operating conditions (e.g. superficial gas velocity and solids circulation rate) on the flow behaviour in the CFB riser. The cold model CFB is shown schematically in Figure 3. The column was 10 m

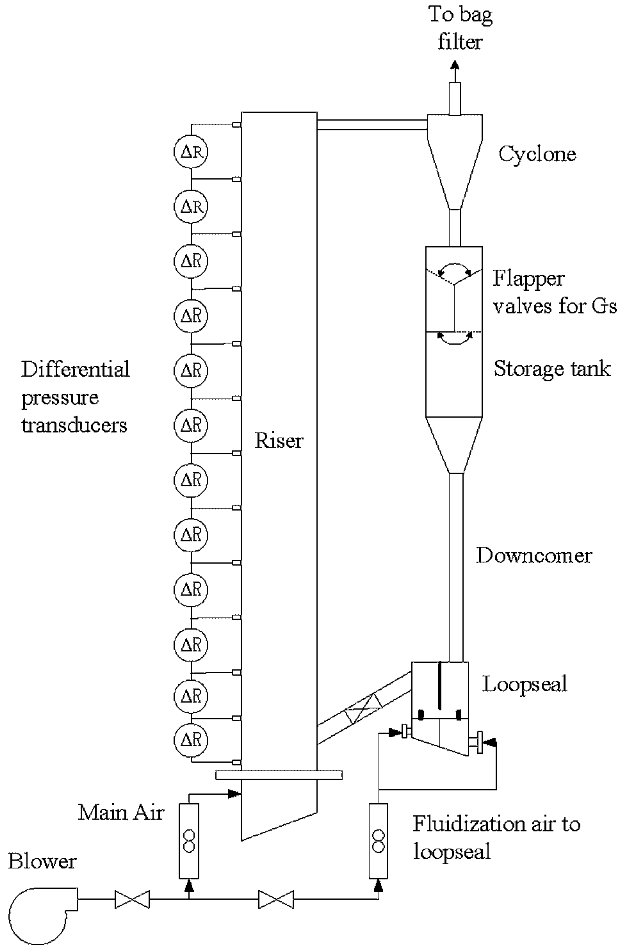

Figure 3. Schematic of a cold model CFB.

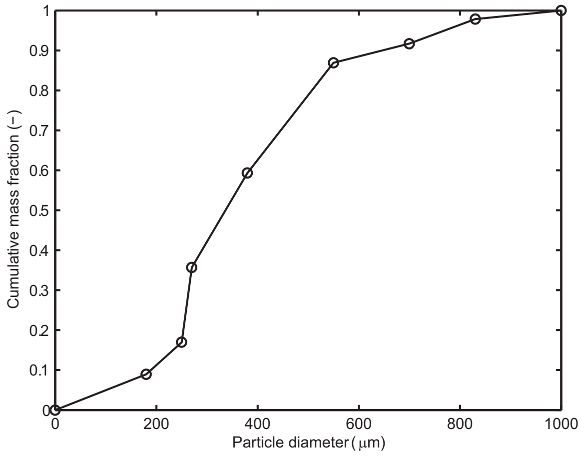

Figure 4. Cumulative size distribution of sawdust.

in height and 280 mm in diameter, and was equipped with 12 differential pressure transducers to measure axial pressure drop along the riser. A reflective optical fibre probe was employed to measure local solids holdup. The effects of operating conditions on axial and radial distributions of solids holdup and particle velocity were analysed. Refer to Miao et al. (2011) for a detailed description about the experimental setup. The bed material used in this study was sawdust with mean diameter $d_p = 400\ \mu m$, particle density $\rho_p = 430\ kg/m^3$ and sphericity $\varphi = 0.35$. Its cumulative size distribution is shown in Figure 4.

In order to estimate the axial profiles of cross-sectional average solids holdup in the riser, 12 OMEGA PX series differential pressure transducers were installed to measure pressure drop along the riser column, as shown in Figure 3. A multi-fibre optical probe was chosen to measure local solids holdup in this study. Local solids concentrations under eight operating conditions were measured at eight radial positions ($r/R = 0.0, 0.143, 0.286, 0.429, 0.571, 0.714, 0.857, 0.929$) on eight axial levels ($z = 0.85, 1.35, 2.35, 3.35, 4.35, 5.35, 6.35, 8.35$ m). Refer to Miao et al. (2011) for more details about the experimental method.

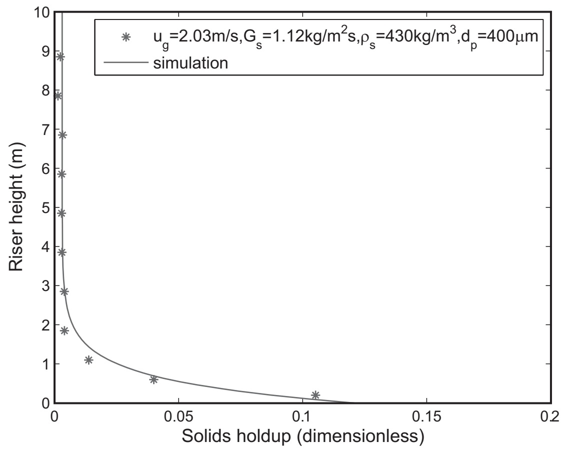

Figure 5. Comparison of axial solids holdup (this work, sawdust as material). [Color figure can be seen in the online version of this article, available at http://onlinelibrary.wiley.com/journal/10.1002/(ISSN)1939-019X]

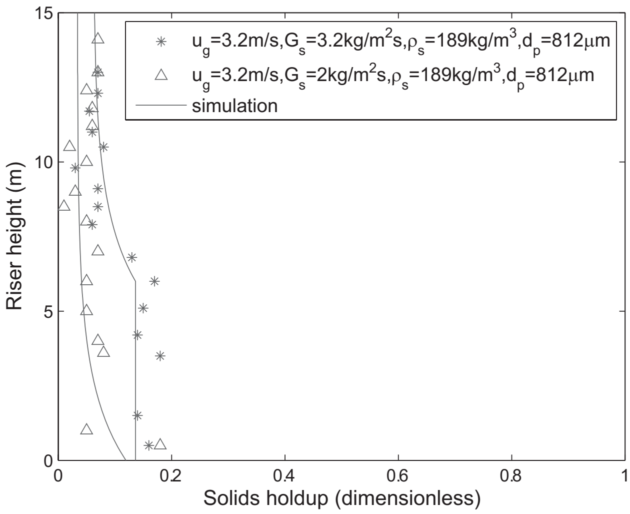

Figure 6. Comparison of axial solids holdup (Mei et al., 2006, cork as material). [Color figure can be seen in the online version of this article, available at http://onlinelibrary.wiley.com/journal/10.1002/(ISSN)1939-019X]

SIMULATION RESULTS AND DISCUSSIONS

Figures 5 and 6 compare the simulated and experimental axial solids holdup profiles for experiments conducted in this work and Mei et al. (2006). Both the simulation and experimental data showed similar trend. The solids concentration was high at the bottom and then decreased gradually due to the acceleration of the solids until it becomes relatively constant at higher level. According to Bai and Kato (1999), at a constant superficial gas velocity, when the solids circulation rate is increased to a value at which much of the solids begin to accumulate at the bottom of the riser, a typical S-shaped solids holdup distribution starts to form. Further increasing solids circulation rate beyond this critical point will have negligible effect on the solids holdup at the dense and dilute regions, although the dense region height continues to increase. This critical point is the so-called saturation carrying capacity of gas, G_s^*. In Figure 6, G_s^* in Mei et al. (2006)

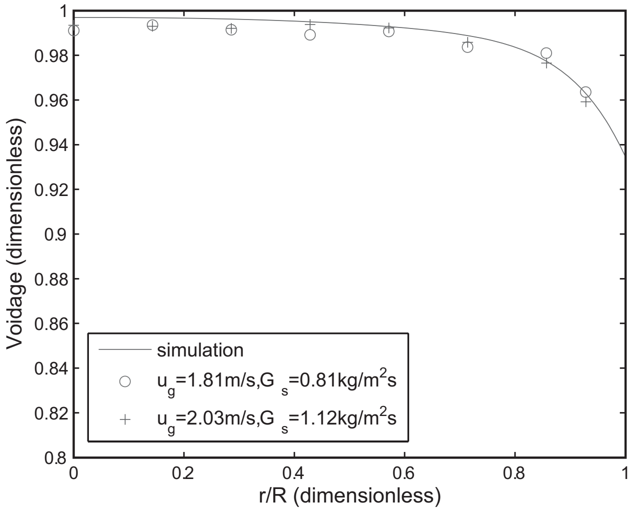

Figure 7. Comparison of radial voidage (this work, $z = 0.85$ m). [Color figure can be seen in the online version of this article, available at http://onlinelibrary.wiley.com/journal/10.1002/(ISSN)1939-019X]

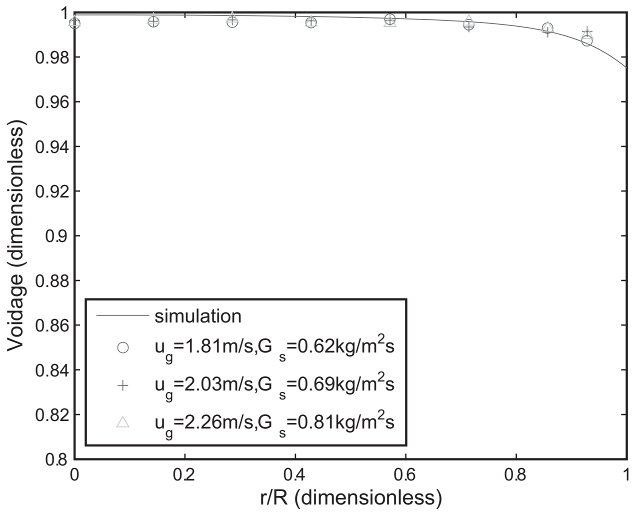

Figure 8. Comparison of radial voidage (this work, $z = 0.85$ m). [Color figure can be seen in the online version of this article, available at http://onlinelibrary.wiley.com/journal/10.1002/(ISSN)1939-019X]

was experimentally identified at 3.1 kg/m^2 s. It can be seen clearly from both the experimental data and the simulation predictions that when $G_s > G_s^*$, a dense region with a constant solids holdup appears at the bottom of the bed, and the axial solids holdup profile is a typical S-shape. The height of the bottom bed depends on particle properties and operating conditions. Generally, Figure 6 shows that the higher the solids circulation rate, the denser the solids holdup in the whole riser and the higher the bottom bed.

Figures 7 and 8 compare experimentally determined radial voidage profiles in this work with simulation predictions. Both the model and the experimental data show that the voidage distribution is flat in the core region and steep close to the wall. The model also demonstrated that the voidage profiles under different operating conditions coincide with each other, provided they have the same average cross-sectional average bed voidage. The good agreement between the experimental data and the predictions indicates that Zhang et al.'s (1991) equation, which was derived based on Group A particles, can be applied to low-density particles as well.

CONCLUSIONS

A mathematical model was developed to estimate axial and radial solids holdup and bed voidage profiles of CFB risers under fast fluidisation conditions. Kunii and Levenspiel's (1991) model was modified and applied to express the vertical solids distribution with some other assumptions. The simple model assumptions based on their detected phenomena were in agreement with the experimental data. Two correlations (Equations 3 and 4) were proposed to predict the solids holdup at dense region and dilute region, respectively. Zhang et al.'s (1991) equation was employed to describe the radial voidage profile in the present study, and it was proven from the comparison results that Zhang et al.'s (1991) equation can be applied to light particles as well. The verification of the simulation results with experimental data showed that reasonable agreements were achieved. Through comparison with experimental data, the proposed model was shown to be able to qualitatively and quantitatively predict the effect of operating variables such as superficial gas velocity and solids circulation rate on the axial and radial solids holdup profiles.

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NOMENCLATURE

a	decay constant
Ar	Archimedes number ($=d_p^3\rho_g g(\rho_p - \rho_g)/\mu_g^2$)
d_p	particle diameter (μm)
D	riser internal diameter (m)
G_s	solids circulation rate (kg/m^2 s)
G_s^*	saturation carrying capacity of gas (kg/m^2 s)
g	acceleration due to gravity (m/s^2)
H_b	dense region height (m)
R	riser radius (m)
r	horizontal distance from axis (m)
u_g	superficial gas velocity (m/s)
u_t	terminal particle velocity (m/s)
z	axial distance from the distributor (m)

Greek Symbols

ε	local bed voidage
ε_s	cross-sectional average solids holdup
ε_{sb}	cross-sectional average solids holdup at dense region
ε_s^*	cross-sectional average solids holdup at dilute region
μ	gas viscosity (Pa s)
ρ_g	gas density (kg/m^3)
ρ_p	particle density (kg/m^3)
φ	sphericity

REFERENCES

Adánez, J., P. Gayán, F. García-Labiano and L. F. de Diego, "Axial Voidage Profiles in Fast Fluidised Beds," Powder Technol. **81**, 259–268 (1994).

Bai, D. and K. Kato, "Quantitative Estimation of Solids Holdups at Dense and Dilute Regions of Circulating Fluidised Beds," Powder Technol. **101**, 183–190 (1999).

Bi, H. T., Y. Jin, Z. Q. Yu and D. R. Bai, "The Radial Distribution of Heat Transfer Coefficient in Fast Fluidised Bed," in "Fluidisation VI," J. R. Grace, L. W. Shemilt and M. A. Bergougnou, Eds., Engineering Foundation, New York (1989), pp. 701–708.

Cabezas-Gómez, L. and F. E. Milioli, "Numerical Study on the Influence of Various Physical Parameters Over the Gas-Solid Two-Phase Flow in the 2D Riser of a Circulating Fluidised Bed," Powder Technol. **132**, 216–225 (2003).

de Diego, L. F., P. Gayán and J. Adánez, "Modelling of the Flow Structure in Circulating Fluidised Beds," Powder Technol. **85**, 19–27 (1995).

Gao, S., G. Zhao, S. Qiu and W. Ma, "Solid Circulating Rate in Fast Fluidised Bed," in "Fluidisation-91," M. Kwauk and D. Kunii, Eds., Science Press, Beijing (1991), pp. 76.

Godfroy, L., G. S. Patience and J. Chaouki, "Radial Hydrodynamics in Risers," Ind. Eng. Chem. Res. **38**, 81–89 (1999).

Grace J. R., A. A. Avidan and T. M. Knowlton, Eds., "Circulating Fluidised Beds," Chapman & Hall Press, London, New York (1997).

Gungor, A. and N. Eskin, “Hydrodynamic Modelling of a Circulating Fluidised Bed,” Powder Technol. **172**, 1–13 (2007).

Harris, B. J. and J. F. Davidson, “Modelling Options for Circulating Fluidised Beds: A Core/Annulus Deposition Model,” in “Circulating Fluidised Bed Technology IV,” A. A. Avidan, Ed., AIChE, New York (1994), pp. 32–39.

Kunii, D. and O. Levenspiel, “Entrainment of Solids From Fluidised Beds,” Powder Technol. **61**, 193–206 (1990).

Kunii, D. and O. Levenspiel, “Fluidisation Engineering,” 2nd ed., Boston, Butterworth-Heinemann (1991).

Li, Y. and M. Kwauk, “The Dynamics of Fast Fluidisation,” in “Fluidisation,” J. R. Grace and J. M. Masten, Eds., New York, Plenum Press (1980), pp. 537–544.

Lu, H. and D. Gidaspow, “Hydrodynamic Simulations of Gas-Solid Flow in a Riser,” Ind. Eng. Chem. Res. **42**, 2390–2398 (2003).

Mathiesen, V., T. Solberg and B. H. Hjertager, “Predictions of Gas/Particle Flow With an Eulerian Model Including a Realistic Particle Size Distribution,” Powder Technol. **112**, 34–45 (2000).

Mei, J. S., E. R. Monazam and L. J. Shadle, “Flow Regime Study of a Light Material in an Industrial Scale Cold Flow Circulating Fluidised Bed,” J. Energy Resour. Technol. **128**, 129–134 (2006).

Miao, Q., C. Wang, C. Wu, X. Yin and J. Zhu, “Fluidisation of Sawdust in a Cold Model Circulating Fluidised Bed: Experimental Study,” Chem. Eng. J. **167**, 335–341 (2011).

Nieuwland, J. J., M. van Sint Annaland, J. A. M. Kuipers and W. P. M. van Swaaij, “Hydrodynamic Modelling of Gas/Particle Flows in Riser Reactors,” AIChE J. **42**, 1569–1582 (1996).

Patience, G. S. and J. Chaouki, “Solids Hydrodynamics in the Fully Developed Region of CFB Risers,” in “Fluidisation VIII (Preprints),” J.-F. Large and C. Laguerie, Eds., Tours, France, Engineering Foundation (1995), pp. 33–40.

Pugsley, T. S. and F. Berruti, “A Predictive Hydrodynamic Model for Circulating Fluidised Bed Risers,” Powder Technol. **89**, 57–69 (1996).

Rhodes, M. J., “Modelling the Flow Structure of Upward-Flowing Gas-Solids Suspensions,” Powder Technol. **60**, 27–38 (1990).

Rhodes, M. J. and D. Geldart, “A Model for the Circulating Fluidised Bed,” Powder Technol. **53**, 155–162 (1987).

Rhodes, M. J., X. S. Wang, H. Cheng, T. Hirama and B. M. Gibbs, “Similar Profiles of Solids Flux in Circulating Fluidised-Bed Risers,” Chem. Eng. Sci. **47**(7), 1635–1643 (1992).

Schlichthaerle, P. and J. Werther, “Axial Pressure Profiles and Solids Concentration Distributions in the CFB Bottom Zone,” Chem. Eng. Sci. **54**, 5485–5493 (1999).

Smolders, K. and J. Baeyens, “Hydrodynamic Modelling of the Axial Density Profile in the Riser of a Low-Density Circulating Fluidised Bed,” Can. J. Chem. Eng. **79**(3), 422–429 (2001).

Sun, B. and D. Gidaspow, “Computation of Circulating Fluidised-Bed Riser Flow for the Fluidisation VIII Benchmark Test,” Ind. Eng. Chem. Res. **38**, 787–792 (1999).

Svensson, A., F. Johnsson and B. Leckner, “Bottom Bed Regimes in A Circulating Fluidised Bed Boiler,” Int. J. Multiphase Flow **22**(6), 1187–1204 (1996).

Wang, J., W. Ge and J. Li, “Eulerian Simulation of Heterogeneous Gas-Solid Flows in CFB Risers: EMMS-Based Sub-Grid Scale Model With a Revised Cluster Description,” Chem. Eng. Sci. **63**, 1553–1571 (2008).

Wen, C. Y. and L. H. Chen, “Fluidised Bed Freeboard Phenomena: Entrainment and Elutriation,” AIChE J. **28**(1), 117–128 (1982).

Werther, J., “Fluid mechanics of large-scale CFB units,” in “Circulating Fluidised Bed Technology IV,” Proceedings of the Fourth International Conference on Circulating Fluidised Beds, A. A. Avidan, Eds., Pennsylvania, U.S.A. (1994), pp. 1–14.

Xu, G., G. Sun and S. Gao, “Estimating Radial Voidage Profiles for All Fluidisation Regimes in Circulating Fluidised Bed Risers,” Powder Technol. **139**, 186–192 (2004).

Yang, G. L., Z. Huang and L. Z. Zhao, “Radial Gas Dispersion in a Fast Fluidised Bed,” in “Fluidisation IV,” D. Kunii and R. Toei, Eds., New York, Engineering Foundation (1984), pp. 145–152.

Zhang, W., Y. Tung and F. Johnsson, “Radial Voidage Profiles in Fast Fluidised Beds of Different Diameters,” Chem. Eng. Sci. **46**, (12), 3045–3052 (1991).

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